On hyperholomorphic polynomials for the Cauchy-Riemann and the Dirac operators of Clifford analysis.

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Let $\mathcal{D} := \sum_{k=0}^{n} e_k \frac{\partial}{\partial x_k}$ be the Cauchy-Riemann (CR) operator of Clifford analysis and let $\widetilde{\mathcal{D}} := \sum_{k=1}^{n} e_k \frac{\partial}{\partial x_k}$ be the corresponding Dirac operator. As was shown first by Fueter for quaternionic analysis, the functions $\zeta_\ell(x) := x_\ell - x_0 e_\ell, \quad \ell \in \{1, \ldots, n\}$, form a basis of CR-hyperholomorphic homogeneous polynomials of degree one, and they generate a basis of all CR-hyperholomorphic polynomials thus having consequences in constructing the Taylor series and rational functions for the Cauchy-Riemann operator.

The functions ζ_ℓ cannot serve for Dirac-hyperholomorphic functions but if one introduces the new functions $\widetilde{\zeta}_k := x_1 e_1 e_k + x_k, \quad k \in \{2, \ldots, n\}$, they form already a basis of Dirac-hyperholomorphic functions and all the statements about arbitrary CR-hyperholomorphic polinomials, Taylor series and rational functions, have their counterparts in the context of Dirac-hyperholomorphy.

Since any Dirac-hyperholomorphic function can be identified with a CR-hyperholomorphic one which does not depend on x_0, we can expect that all the objects considered in the Dirac theory are particular cases of their CR-analogs which is not so even on the level of the functions ζ_ℓ and $\widetilde{\zeta}_k$. Thus the question arises whether it is possible to construct a similar theory for the CR-operator but in such a way that for CR-hyperholomorphic functions which do not depend on x_0, it would turn directly into the theory for the Dirac operator.

The aim of the talk is to expose a positive answer and to explain a number of new phenomena arising here. The talk is based on joint results with D. Alpay, F.M. Correa-Romero and M.E. Luna-Elizarrarás.

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